# Introduction to Network Science

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Topic Today

### IDENTIFY INFLUENTIAL **SPREADERS** Influence Analysis of Nodes in **Complex Networks**



#### Introduction

Identifying the most efficient spreaders in a network is an important step towards optimizing the use of available resources and ensuring the more efficient spread of information.



#### Main Methods





Historically first and conceptually simplest is degree centrality, which is defined as the number of links incident upon a node. In the case of a directed network (where ties have direction), we usually define two separate measures of degree centrality, namely indegree and outdegree

The degree centrality of a vertex v, for a given graph G:=(V, E), with |V| vertices and |E| edges, is defined as

 $C_D(v) = \deg(v)$ 

After normalization, the degree centrality is defined as

$$C_D(v)' = \frac{\deg(v)}{|V| - 1}$$

Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes

The betweenness centrality of a vertex v, for a given graph G := (V, E), with |V| vertices and |E| edges, is defined as

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where  $\sigma_{st}$  is total number of shortest paths from node *s* to node *t* and  $\sigma_{st}(v)$  is the number of those paths that pass through v.



Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes

After normalization, the betweenness centrality can be represented as

$$C_B(v) = \left(\sum_{\substack{s \neq v \neq t \in V}} \frac{\sigma_{st}(v)}{\sigma_{st}}\right) / (|V| - 1)(|V| - 2) \quad \text{(directed)}$$

$$C_B(v) = 2 \times \left(\sum_{\substack{s \neq v \neq t \in V}} \frac{\sigma_{st}(v)}{\sigma_{st}}\right) / (|V| - 1)(|V| - 2) \quad \text{(undirected)}$$



The farness of a node is defined as the sum of its distances from all other nodes, and its closeness was defined as the reciprocal of the farness.

The closeness centrality of a vertex  $\mathcal{X}$ , for a given graph G := (V, E), with |V| vertices and |E| edges, is defined as

$$C(x) = \frac{1}{\sum_{y} d(y, x)}$$



**The k-shell decomposition:** Nodes are assigned to k shells according to their remaining degree, which is obtained by successive pruning of nodes with degree smaller than the  $k_s$  value of the current layer.

We start by removing all nodes with degree k D1. After removing all the nodes with k D1, some nodes may be left with one link, so we continue pruning the system iteratively until there is no node left with k D1 in the network. The removed nodes, along with the corresponding links, form a k shell with index kS D1. In a similar fashion, we iteratively remove the next k shell, kS D2, and continue removing higher-k shells until all nodes are removed. As a result, each node is associated with one kS index, and the network can be viewed as the union of all k shells. The resulting classification of a node can be very different than when the degree k is used.



**The k-shell decomposition:** Nodes are assigned to k shells according to their remaining degree, which is obtained by successive pruning of nodes with degree smaller than the  $k_s$  value of the current layer.









$$M(k_{\rm S},k) = \sum_{i \in \Upsilon(k_{\rm S},k)} \frac{M_i}{N(k_{\rm S},k)}$$

where  $\Upsilon(k_{\rm S},k)$  is the union of all  $N(k_{\rm S},k)$  nodes with  $(k_{\rm S},k)$  values.



The quantitative analysis: The k-shell index predicts the outcome of spreading more reliably than the degree k or the betweenness centrality  $C_B$ 





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#### WE ARE GROUP 2

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## THANK YOU

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